Objective: TSWBAT find the number of permutations

Vocabulary

1. A **permutation** is an arrangement of a set of objects in a particular order.

2. **Factorial (p. 596)** The product of all positive integers less than or equal to a number is a factorial.

EXAMPLE 2 Using the Counting Principle Real World

Awards At a school awards ceremony, the principal will present awards to seven students. How many different ways can the principal give out the awards?

There are seven ways to give out the first award, six ways to give out the second, and so on.

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5{,}040$ ← Use the counting principle.

There are 5,040 different ways to give out the awards.

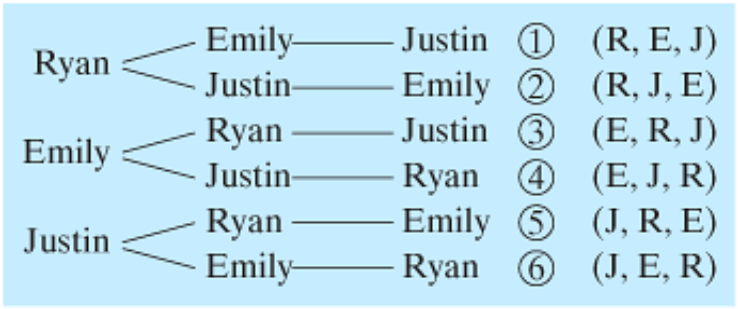
EXAMPLE **Permutations Using Factorials** Real World

Music Many CD players can vary the order in which songs are played. Your favorite CD has eight songs. Find the number of orders in which the songs can be played.

$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$ ← Simplify.

The songs can be played in 40,320 different orders or permutations.

Gym In how many ways can Ryan, Emily, and Justin line up in gym class?



Ryan, Emily, and Justin can line up in six different ways. This means that there are six permutations.

Key Concepts

Permutation Notation

The expression ${}_nP_r$ represents the number of permutations of n objects chosen r at a time.

Example ${}_{25}P_2 = 25 \cdot 24 = 600$

25 objects groups of 2 (two factors)

A class of 25 students must choose a president and a vice president. There are 25 possible choices for the president. Then there are 24 possible choices for the vice president. So, there are $25 \cdot 24$ permutations for choosing a president and a vice president from 25 students. You can write this as ${}_{25}P_2$.

4 EXAMPLE Using Permutation Notation

Simplify ${}_{15}P_3$.

$${}_{15}P_3 = 15 \cdot 14 \cdot 13 \quad \leftarrow \text{Write the notation as a product of 3 factors, starting with 15.}$$
$$= 2{,}730 \quad \leftarrow \text{Simplify.}$$

There are 2,730 permutations of 15 items chosen 3 at a time.

EXAMPLE **Real-World Problem Solving**

Science Fair At a school science fair, ribbons are given for first, second, third, and fourth place. There are 20 exhibits in the fair. How many different arrangements of four winning exhibits are possible?

Use a calculator.

Enter 20. Find the PRB menu. Select ${}_nP_r$.
Enter 4. The display shows $116,280$.

There are 116,280 different arrangements of four winning exhibits.

In how many ways can you pick a football center and quarterback from 6 players who try out?

Art, Becky, Carl, Denise, and Ed all want to go to the concert. However, there are only 3 tickets. How many ways can they choose the 3 who get to go to the concert?

How many 3-letter codes can be made from A, B, C, D, E, F, G, H with no repeating letters?

For a meeting agenda, in how many ways can you schedule 3 speakers out of 10 people who would like to speak?

Numbers are to be formed using the digits 1, 2, 3, 4, 5, and 6. No digit may be repeated.

How many two-digit numbers can be formed? _____

How many three-digit numbers can be formed? _____

How many four-digit numbers can be formed? _____

How many five-digit numbers can be formed? _____

How many six-digit numbers can be formed? _____

Additional Examples

1. In how many ways can four people form a line? [redacted]
2. Suppose you have six invitations to write. In how many different sequences can you write them? [redacted]
3. A CD has nine songs. In how many different orders could you play these songs? [redacted]
4. Simplify ${}_{11}P_5$. [redacted]
5. In a spelling contest, trophies are given for first, second, and third places. There are 15 finalists in the contest. How many different arrangements are possible for the winners of the trophies? [redacted]